tions to be satisfied for the third-order ordinary differential equations (9) and (10) are

$$\left.\begin{array}{l} \left.\begin{array}{l} dH_\phi/dr = -(A\varkappa/\nu)\omega_2 \\ H_\phi = 0 \end{array}\right\} \text{at } r = b \\ \left.\begin{array}{l} dH_z/dr = -U(A\varkappa/\nu b) \\ H_z = 0 \end{array}\right\} \text{at } r = b \\ [(\varkappa R - 1)H_\phi - a(dH_\phi/dr)] = (A\varkappa/\nu)a\omega_1 \text{ at } r = a \\ [\varkappa RH_z - a(dH_z/dr)] = 0 \text{ at } r = a \end{array}\right\} \quad (11)$$

The expressions for V_ϕ, H_ϕ and V_z, H_z are given by

$$(A\varkappa/\nu)V\phi = A_1\varkappa R(1/r) + A_2(\varkappa R - 1 - \lambda_1)r^{\lambda_1} + A_3(\varkappa R - 1 - \lambda_2)r^{\lambda_2} \quad (12)$$

$$H\phi = A_1(1/r) + A_2r^{\lambda_1} + A_3r^{\lambda_2} \quad (13)$$

where

$$\lambda_1,\lambda_2 = \tfrac{1}{2}[R(1 + \varkappa) \pm \{R^2(1 + \varkappa)^2 - 4(R + 1)(\varkappa R - 1) + 4M^2\}^{1/2}] \quad (14)$$

$$(A\varkappa/\nu)V_z = \varkappa RB_1 + (\varkappa R - \alpha_1)B_2r^{\alpha_1} + (\varkappa R - \alpha_2)B_3r^{\alpha_2} + (\varkappa R - 2)Kr^2 \quad (15)$$

$$H_z = B_1 + B_2r^{\alpha_1} + B_3r^{\alpha_2} + Kr^2 \quad (16)$$

$$\left.\begin{array}{l} \alpha_1,\alpha_2 = \tfrac{1}{2}[R(1 + \varkappa) \pm \{R^2(1 - \varkappa)^2 + 4M^2\}^{1/2}] \\ \text{where} \\ K = (AP\varkappa/\rho\nu^2)[(R - 2)(\varkappa R - 2) - M^2]^{-1} \end{array}\right\} \quad (17)$$

and A's and B's are constants of integration. Their values satisfying the boundary conditions (11) become

$$A_1 = (A\varkappa/\nu)(1/c)[\omega_1(\lambda_2 - \lambda_1) \times (a^2b^{1+\lambda_1}b^{1+\lambda_2} - \varkappa Rb^2b^{1+\lambda_1}b^{1+\lambda_2}) + \omega_2\{(\lambda_2 - \lambda_1)\varkappa Rb^2b^{1+\lambda_1}b^{1+\lambda_2} + (\varkappa R - 1 - \lambda_2) \times b^2a^{1+\lambda_2}b^{1+\lambda_1} - (\varkappa R - 1 - \lambda_1)b^2a^{1+\lambda_1}b^{1+\lambda_2}\}] \quad (18)$$

$$A_2 = (A\varkappa/\nu)(1/c)[\omega_1(1 + \lambda_2)(a^2b^{1+\lambda_2} - \varkappa Rb^2b^{1+\lambda_2}) + b^2\omega_2\{\lambda_2\varkappa Rb^{1+\lambda_2} + (\varkappa R - 1 - \lambda_2)a^{1+\lambda_2}\}] \quad (19)$$

$$A_3 = (A\kappa/\nu)(1/c)[\omega_1(1 + \lambda_1) \times (\varkappa Rb^2b^{1+\lambda_1} - a^2b^{1+\lambda_1}) - b^2\omega_2\{\lambda_1\varkappa Rb^{1+\lambda_1} + (\varkappa R - 1 - \lambda_1)a^{1+\lambda_1}\}] \quad (20)$$

where

$$C = \varkappa Rb^{1+\lambda_1}b^{1+\lambda_2}(\lambda_1 - \lambda_2) + (1 + \lambda_2)(\varkappa R - 1 - \lambda_1)a^{1+\lambda_1}b^{1+\lambda_2} - (1 + \lambda_1)(\varkappa R - 1 - \lambda_2)a^{1+\lambda_2}b^{1+\lambda_1}$$

and

$$B_1 = \frac{Kb^2}{D}\left[(\alpha_2 - \alpha_1)\left\{(\varkappa R - 2)(1 - \theta^2) - \frac{A\varkappa}{K\nu b^2}U\right\} - \left(\frac{A\varkappa U}{K\nu b^2} + 2\right)\{(\varkappa R - \alpha_1)(1 - \theta^{\alpha_1}) - (\varkappa R - \alpha_2)(1 - \theta^{\alpha_2})\}\right] \quad (21)$$

$$B_2 = -\frac{K}{Db^{\alpha_1}}\left[\alpha_2(1 - \theta^2)(\varkappa R - 2) - \alpha_2\frac{A\varkappa U}{K\nu} - \left(\frac{A\varkappa U}{K\nu} + 2b^2\right)(1 - \theta^{\alpha_2})(\varkappa R - \alpha_2)\right] \quad (22)$$

$$B_3 = -\frac{K}{Db^{\alpha_2}}\left[(\varkappa R - \alpha_1)(1 - \theta^{\alpha_1})\left(\frac{A\varkappa U}{K\nu} + 2b^2\right) - b^2\alpha_1\left\{(\varkappa R - 2)(1 - \theta^2) - \frac{A\varkappa U}{K\nu b^2}\right\}\right] \quad (23)$$

where

$$D = \alpha_2(\varkappa R - \alpha_1)(1 - \theta^{\alpha_1}) - \alpha_1(1 - \theta^{\alpha_2})(\varkappa R - \alpha_2)$$
$$\theta = a/b$$

The transverse field given by Eqs. (12) and (13) reduces to the one obtained by Ramamoorthy[1] for the case of zero suction velocity at the walls ($R = 0$). The axial field given by Eqs. (15) and (16) agrees with the solution of Jain and Mehta.[7]

In the absence of magnetic field $A = 0$, $M = 0$, $\varkappa = 0$, Eqs. (4) and (5) become exclusive equations in the dependent variables V_ϕ and V_z, respectively. The rotational field reduces to one obtained by Schlichting, and the translational field agrees with Dunwoody[5] and Mehta's[6] results. The field due to simultaneous rotation of the two cylinders and the translation of the outer can be obtained by superposition of the two. It must be noted that the case without the presence of the magnetic field cannot be obtained directly from the solution (12) to (23) by putting $A = 0$, $M = 0$, $\varkappa = 0$. This is accounted for by the fact that, in the case of the hydromagnetic problem, the velocity field is obtained from Eqs. (6) and (7) only after a nonzero magnetic field as a solution of Eqs. (9) and (10) has been determined.

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Thermodynamic Properties of High-Temperature Helium

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Nomenclature

c_2 = second radiation constant = hc/k
E_L = term value of the Lth electronic level, cm^{-1}
f_t = partition function
g_L = statistical weighting function of the Lth electronic level
h = Planck's constant
I_b = term value of the energy required to ionize the parent atom to the bth degree of ionization, cm^{-1}
k = Boltzmann constant
m_{He} = mass of helium
m_e = mass of electron
T = temperature, °K
V = volume of mixture based on one mole at standard conditions
ρ = density

Subscript

0 = standard conditions (see Table 1)

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Introduction

CURRENT and future work in the field of plasma physics requires thermodynamic data to extremely high temperatures. Results for air,[1] nitrogen,[2] argon,[3] and hydrogen[4] have recently been published over a wide range of densities for temperatures up to 100,000°K. It was deemed desirable to calculate the thermodynamic properties of helium over this same range of temperature and density. The purpose of this note is to present a sampling of the computer data for temperatures and densities beyond the range previously published[5, 6] in tabular form. Thus, data are given for a temperature range from 4000° to 100,000°K for density ratios of 0.1, 1, 10, and 100 and for temperatures greater than 50,000°K for density ratios less than 0.1. However, the results in graphical form are available for temperatures from 2000° to 35,000°K for density ratios of 10^{-10} to 10^{2} and temperatures from 2000° to 100,000°K for density ratios of 10^{-6} to 100^{7}.

Table 1 Cutoff criteria

ρ/ρ_0	n^* (Bohr)	n^* (Meghreblian)
10^2	3	2
10	4	3
1	6	4
10^{-1}	9	6
10^{-2}	13	9

Method of Computation

The equations used in the computations are well-known and are given in detail elsewhere,[1–7] therefore, the method will only be outlined here. The molar concentrations were obtained by solving the chemical reaction equations by the method of equilibrium constants under the conditions of constant mass of the system and charge neutrality. The equilibrium constants were computed from the partition functions of the various species of the system. The system under discussion consisted of HeI, HeII, HeIII, and electrons.

The partition function for a monatomic particle is given by

$$f_b = f_b{}^t \exp(-c_2 I_{b-1}/T)\ \Sigma_L g_L \exp(-c_2 E_L/T) \tag{1}$$

where

$$f_i{}^t = V_0\, \rho_0/\rho [2\pi m_i kT/h^2]^{3/2} \tag{2}$$

is identified as the translational partition function, and b denotes the degree of ionization of the particle. In the pres-

Table 2 Thermodynamic properties

ρ/ρ_0	$T \times 10^{-3}$°K	P/P_0	H/RT_0	S/R	C_{He}	C_{He^+}	$C_{\mathrm{He}^{++}}$	C_{e^-}
10^2	4	1.39*E* 3[a]	3.47*E* 1	1.44*E* 1	1.00	...	...	3.17*E*-16
	6	2.08 3	5.21 1	1.50 1	1.00	...	...	6.26 -11
	8	2.78 3	6.94 1	1.55 1	1.00	...	...	2.97 -8
	10	3.47 3	8.68 1	1.58 1	1.00	...	...	1.24 -6
	15	5.20 3	1.30 2	1.64 1	1.00	1.95*E*-4	...	1.95 -4
	20	6.96 3	1.77 2	1.69 1	9.97*E*-1	2.61 -3	...	2.61 -3
	25	8.79 3	2.35 2	1.74 1	9.87 -1	1.28 -2	...	1.28 -2
	30	1.08 4	3.21 2	1.81 1	9.63 -1	3.72 -2	...	3.72 -2
	35	1.31 4	4.52 2	1.90 1	9.21 -1	7.89 -2	...	7.89 -2
	40	1.58 4	6.31 2	2.02 1	8.65 -1	1.35 -1	...	1.35 -1
	50	2.18 4	1.05 3	2.24 1	7.41 -1	2.59 -1	...	2.59 -1
	60	2.83 4	1.41 3	2.40 1	6.39 -1	3.60 -1	3.72*E*-4	3.61 -1
	70	3.49 4	1.69 3	2.50 1	5.63 -1	4.35 -1	2.08 -3	4.39 -1
	80	4.17 4	1.95 3	2.58 1	5.04 -1	4.88 -1	7.54 -3	5.03 -1
	90	4.89 4	2.22 3	2.64 1	4.54 -1	5.26 -1	2.01 -2	5.66 -1
	100	5.67 4	2.54 3	2.71 1	4.08 -1	5.49 -1	4.23 -2	6.34 -1
10	4	1.39 2	3.47 1	1.67 1	1.00	...	...	1.00 -15
	6	2.08 2	5.21 1	1.73 1	1.00	...	...	1.98 -10
	8	2.78 2	6.94 1	1.78 1	1.00	...	...	9.38 -8
	10	3.47 2	8.68 1	1.81 1	1.00	...	...	3.92 -6
	15	5.21 2	1.31 2	1.87 1	1.00	6.18 -4	...	6.18 -4
	20	7.00 2	1.83 2	1.93 1	9.92 -1	8.23 -3	...	8.23 -3
	25	9.02 2	2.67 2	2.01 1	9.60 -1	3.98 -2	...	3.98 -2
	30	1.16 3	4.15 2	2.14 1	8.87 -1	1.13 -1	...	1.13 -1
	35	1.49 3	6.38 2	2.30 1	7.72 -1	2.28 -1	...	2.28 -1
	40	1.90 3	9.11 2	2.48 1	6.35 -1	3.65 -1	...	3.65 -1
	50	2.78 3	1.42 3	2.76 1	4.00 -1	6.00 -1	3.46 -4	6.00 -1
	60	3.62 3	1.79 3	2.90 1	2.65 -1	7.31 -1	3.69 -3	7.39 -1
	70	4.44 3	2.10 3	3.00 1	1.93 -1	7.88 -1	2.00 -2	8.28 -1
	80	5.32 3	2.47 3	3.12 1	1.48 -1	7.85 -1	6.64 -2	9.18 -1
	90	6.36 3	2.97 3	3.25 1	1.16 -1	7.32 -1	1.53 -1	1.03
	100	7.56 3	3.57 3	3.40 1	8.90 -2	6.44 -1	2.67 -1	1.18
1.0	4	1.39 1	3.47 1	1.90 1	1.00	...	...	3.18 -15
	6	2.08 1	5.21 1	1.96 1	1.00	...	...	6.26 -10
	8	2.78 1	6.94 1	2.00 1	1.00	...	...	2.97 -7
	10	3.47 1	8.68 1	2.04 1	1.00	...	...	1.24 -5
	15	5.22 1	1.32 2	2.10 1	9.98 -1	1.95 -3	...	1.95 -3
	20	7.12 1	2.04 2	2.19 1	9.74 -1	2.58 -2	...	2.58 -2
	25	9.72 1	3.65 2	2.36 1	8.80 -1	1.20 -1	...	1.20 -1
	30	1.37 2	6.63 2	2.63 1	6.86 -1	3.14 -1	...	3.14 -1
	35	1.88 2	1.04 3	2.92 1	4.49 -1	5.51 -1	...	5.51 -1
	40	2.42 2	1.38 3	3.13 1	2.60 -1	7.40 -1	1.05 -4	7.40 -1
	50	3.32 2	1.77 3	3.33 1	9.15 -2	9.05 -1	3.43 -3	9.12 -1
	60	4.14 2	2.09 3	3.46 1	4.47 -2	9.21 -1	3.47 -2	9.90 -1
	70	5.16 2	2.62 3	3.65 1	2.73 -2	8.20 -1	1.53 -1	1.13
	80	6.51 2	3.43 3	3.91 1	1.72 -2	6.23 -1	3.60 -1	1.34
	90	8.01 2	4.29 3	4.15 1	9.92 -3	4.16 -1	5.74 -1	1.56
	100	9.47 2	5.02 3	4.32 1	5.28 -3	2.60 -1	7.34 -1	1.73

ent calculations, the reaction energies, weighting factors, and observed term values were taken from Moore.[8]

In order to determine the maximum allowable principal quantum number n^*, which specifies the terms contained in the electronic partition function of the neutral atom, two methods were considered. One approach was to assume that when the radius of the orbit of the excited electrons corresponded to one-half of the average distance between the atoms, the sum would be terminated. The Bohr model was used to calculate this radius. The second approach made use of a simplified version of the theory of Meghreblian.[9] Both of these methods resulted in density dependent cutoff criteria that appear in Table 1. Calculations were performed with these values of n^*. It was determined[10] that deviations of no more than 2% in the thermodynamic properties, except for density ratios of 100, were introduced by the use of $n^* = 4$. Since the methods of selecting n^* are of less accuracy than this, all of the calculations were performed by using $n^* = 4$. For densities less than $\rho/\rho_0 = 0.1$, the properties are independent of n^* for $3 \leqq n^* \leqq 21$. The same value of n^* was selected for calculating the partition function of HeII since the increased electric field due to the changed particles would tend to offset the decreased radius of the ion with respect to the atom.

Discussion of Results

The calculated data are presented in Table 2. The table contains values of pressure, enthalpy, and species concentrations as a function of temperature for various values of density ratios ranging from 10^{-6} to 10^2. For density ratios less than 0.1, data are not presented for temperature less than 50,000°K, because the agreement between the present results and that of Refs. 5 and 6 would make presentation of that data a needless duplication.

In a work of this type, it is inevitable that certain weaknesses in the theory appear. The greatest weaknesses in the present work are associated with the deviation from a perfect gas and the selection of some cutoff criterion to be applied to the summation of the various energy level contributions contained in the electronic partition function. An investigation[10] of the effects of cutoff at various principal quantum numbers was conducted, and a summary of the results is stated in the foregoing.

One of the original assumptions made in the derivation of the thermodynamic equations concerned the absence of interparticle forces. This assumption is obviously open to question when the degree of ionization becomes large and ionic forces become important, since these forces are possibly of a long-range nature. This effect will be most pronounced at

Table 2 (continued)

ρ/ρ_0	$T \times 10^{-3}$°K	P/P_0	H/RT_0	S/R	C_{He}	C_{He^+}	$C_{He^{++}}$	C_{e^-}
10^{-1}	4	1.39E	3.47E 1	2.13E 1	1.00	...	...	1.00 -14
	6	2.08	5.21 1	2.19 1	1.00	...	...	1.98 -9
	8	2.78	6.94 1	2.24 1	1.00	...	...	9.38 -7
	10	3.47	8.68 1	2.27 1	1.00	...	...	3.92 -5
	15	5.24	1.37 2	2.34 1	9.94E-1	6.16E-3	...	6.16E-3
	20	7.49	2.66 2	2.52 1	9.21 -1	7.93 -2	...	7.93 -2
	25	1.16 1	6.19 2	2.91 1	6.68 -1	3.32 -1	...	3.32 -1
	30	1.75 1	1.11 3	3.37 1	3.21 -1	6.79 -1	...	6.79 -1
	35	2.29 1	1.45 3	3.63 1	1.16 -1	8.84 -1	...	8.84 -1
	40	2.72 1	1.63 3	3.73 1	4.35 -2	9.55 -1	1.05E-3	9.58 -1
	50	3.51 1	1.93 3	3.87 1	1.08 -2	9.57 -1	3.24 -2	1.02
	60	4.64 1	2.66 3	4.19 1	4.60 -3	7.64 -1	2.32 -1	1.23
	70	6.24 1	3.81 3	4.63 1	1.98 -3	4.27 -1	5.71 -1	1.57
	80	7.80 1	4.73 3	4.92 1	7.01 -4	1.89 -1	8.11 -1	1.81
	90	9.11 1	5.29 3	5.07 1	2.38 -4	8.14 -2	9.18 -1	1.92
	100	1.03 2	5.67 3	5.15 1	...	3.86 -2	9.61 -1	1.96
10^{-2}	50	3.85	2.44 3	4.63 1	1.05 -3	7.78 -1	2.21 -1	1.22
	60	5.60	3.90 3	5.30 1	2.58 -4	3.12 -1	6.88 -1	1.69
	70	7.08	4.77 3	5.63 1	...	8.38 -2	9.16 -1	1.92
	80	8.26	5.19 3	5.75 1	...	2.48 -2	9.75 -1	1.98
	90	9.34	5.50 3	5.81 1	...	9.12 -2	9.91 -1	1.99
	100	1.04 1	5.77 3	5.86 1	...	4.08 -3	9.96 -1	2.00
10^{-3}	50	4.64 -1	3.63 3	5.83 1	...	3.26 -1	6.74 -1	1.67
	60	6.14 -1	4.61 3	6.27 1	...	4.97 -2	9.50 -1	1.95
	70	7.26 -1	4.98 3	6.39 1	...	9.41 -3	9.91 -1	1.99
	80	8.32 -1	5.26 3	6.46 1	...	2.56 -3	9.97 -1	2.00
	90	9.37 -1	5.52 3	6.51 1	...	9.23 -4	1.00	2.00
	100	1.04	5.78 3	6.56 1	...	4.10 -4	1.00	2.00
10^{-4}	50	5.11 -2	4.34 3	6.87 1	...	5.32 -2	9.47 -1	1.95
	60	6.24 -2	4.73 3	7.01 1	...	5.32 -3	9.95 -1	1.99
	70	7.29 -2	5.00 3	7.09 1	...	9.52 -4	9.99 -1	2.00
	80	8.33 -2	5.26 3	7.15 1	...	2.57 -4	1.00	2.00
	90	9.37 -2	5.52 3	7.20 1	...	...	1.00	2.00
	100	1.04 -1	5.78 3	7.25 1	...	...	1.00	2.00
10^{-5}	50	5.20 -2	4.47 3	7.62 1	...	5.72 -3	9.94 -1	1.99
	60	6.25 -3	4.74 3	7.71 1	...	5.36 -4	1.00	2.00
	70	7.29 -3	5.00 3	7.78 1	...	...	1.00	2.00
	80	8.33 -3	5.26 3	7.84 1	...	...	1.00	2.00
	90	9.37 -3	5.52 3	7.89 1	...	...	1.00	2.00
	100	1.04 -2	5.78 3	7.94 1	...	...	1.00	2.00
10^{-6}	50	5.20 -4	4.48 3	8.32 1	...	5.77 -4	9.99 -1	2.00
	60	6.25 -4	4.74 3	8.40 1	...	...	1.00	2.00
	70	7.29 -4	5.00 3	8.47 1	...	...	1.00	2.00
	80	8.33 -4	5.26 3	8.53 1	...	...	1.00	2.00
	90	9.37 -4	5.52 3	8.58 1	...	...	1.00	2.00
	100	1.04 -3	5.78 3	8.63 1	...	...	1.00	2.00

[a] 1.39E3 denotes 1.39×10^3.

higher pressure when the time between collisions is decreased. As the density is decreased, the time that a particle is acted upon by another, compared to the time between collisions, will decrease in a manner proportional to the density. Therefore, the interparticle forces may probably be neglected for the lower densities. Magee and Heller[11] used a Debye-Huckel ionic solution in estimating the effect of the ionic forces on helium, among other plasmas, up to 50,000°K and found that this correction amounted to only a few percent at most. It is felt that the same results apply to a higher temperature, probably with a somewhat higher error. However, at this time there exists, to the knowledge of the authors, no adequate expression for these forces which would allow one to compensate for their effects with an accuracy justifying the additional complications.

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A Semitoroidal Reflex Discharge as a Propulsion Device

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A REFLEX discharge consists essentially of two cathodes situated at each end of a cylindrical hollow anode. An axial magnetic field confines the plasma that is formed by ionizing electrons emitted by the cathodes and accelerated by the anode. Because of the symmetrical variation of the potential, the electrons are first accelerated by the anode, cross the anode region (almost equipotential), and then are reflected by the second cathode. The electrons may oscillate back and forth between cathodes several times and can lose an important part of their energy in ionizing collisions with neutrals before falling finally to the anode. The ions formed in the preceding collisions are accelerated by the negative potential of one or the other of the cathodes. If Vd is the voltage of the discharge, the energy acquired by the ion as it falls to the cathode is on the order of eV_d. If one of the cathodes has a ring shape,[1] the ions can travel through it. The beam of ions extracted is automatically neutralized by electrons that have sufficient energy to cross the potential barrier of the cathode and thus accompany the ions outside the discharge region.

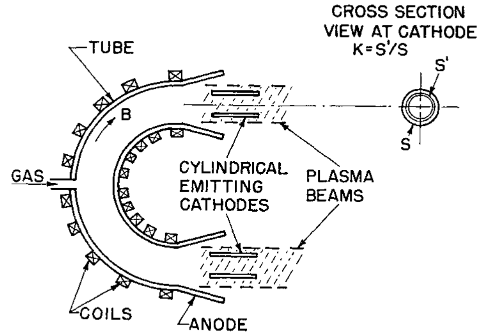

Fig. 1 Diagram of semitoroidal discharge (the tube may be of insulating material or may be metal and be at anode potential).

The electrons that do not accompany the ions are reflected by the voltage drop that occurs in the ring cathode region, and they behave exactly as in an ordinary reflex discharge. Therefore, the high degree of ionization provided by the combined electric and magnetic field of the reflex discharge is not affected.

Electrical efficiency η_E is defined as the ratio of the kinetic energy of the extracted plasma beam to the total electrical energy expended in the system

$$\eta_E = P_C/P_E$$

where

P_C = kinetic power of beam
P_E = electrical power expended

If A is the number of ionizing collisions produced by one electron emitted by the cathode and accelerated by the cathode-anode potential, and K is the number of ions lost (falling on cathodes) divided by the total number of ions produced, then it can be shown[2] that

$$\eta_E = A(1 - K)/(1 + A)$$

One can see immediately that if $A \to \infty$ (i.e., if the ionization energy becomes negligible), $\eta_E \to 1 - K$. For a linear discharge terminated at one end by the emitting cathode, $K \geq 0.5$, giving $\eta_E \leq 50\%$. This results from the fact that the ions formed in the anode region have equal chance to be extracted or to fall on the emitting cathode where their kinetic energy is lost. Then it seemed at first that the efficiency of a reflex discharge as a propulsion device was basically limited to 50%.

However, the addition of a mirror magnetic field forces the electrostatic potential to be asymmetric.[3] As a result, the ions are directed toward the exit rather than toward the emitting cathode.

A reflex discharge having a semitoroidal shape, and using two cylindrical emitting cathodes, offers another possibility to increase the efficiency above 50%. The diagram of the discharge is given in Fig. 1.

In that case, K has the lower theoretical value of $K = S'/S$ where S is the area of the torus cross section (perpendicular to the torus axis), and S' is the area of the cathode cross section (perpendicular to the axis). A typical value of K is $K = 0.1$. The ionization efficiency of the electrons is conveniently

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